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Zeroes of the $e^+e^- \to \bar{f}f$ cross section and search for new physics.

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Abstract:

We suggest returning to a different presentation of the $e^+e^- \to \bar{f}f$ data off the Z peak, with the hope of using zeroes of specific amplitudes to enhance the sensitivity to new physics.

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As well known the electron-positron annihilation into light fermions $e^+e^- \rightarrow \bar{f}f$ is best described at tree level in terms of the helicity amplitudes, which do not interfere in the massless limit.

To the amplitudes $A^f_{ij}(i,j = L, R)$ with i referring to the polarisation of the initial electron, and j to that of the final fermion correspond the cross sections given by σ_{ij}, and in the tree level massless limit:

$$\sigma = \frac{1}{2}(\sigma_{LL} + \sigma_{RR} + \sigma_{LR} + \sigma_{RL}) \tag{1}$$

with $\frac{1}{2}$ corresponding to the spin averaging for unpolarized initial beams. Due to $\gamma - Z$ interference, we expect each of these terms to vanish (up to the Z width) for a specific value of the center of mass energy $\sqrt{s}$.

For the known fermions, we observed that σ_{LL} and σ_{RR} both vanish for $\sqrt{s} < m_Z$, while the zeros in $\sigma_{LR} + \sigma_{RL}$ are found at $\sqrt{s} > m_Z$ with the exception of σ^d_{RL}, for which the photon coupling is insufficient to achieve a total destructive interference. Instead of the usual analysis in terms of forward-backward asymmetries A_{FB}, we advocate thus to turn to the above decomposition using appropriate filters. Separate studies of $(\sigma_{LL} + \sigma_{RR})$ and $(\sigma_{LR} + \sigma_{RL})$, or of their ratios, close to the γZ interference minima allow to:

- exhibit in a self-explanatory way the $\gamma - Z$ interference.
- test the position and height of the minima
- minimize the background for the observation of possible new scalar interactions (which don't interfere with the Z, and don't benefit in any case from the Z peak enhancement)
- understand readily the different impact of extra gauge bosons (Z'), family-universal or otherwise .

The possibility also exists, at least at the theoretical level, to enhance the up- or down-quark content of the final state (in particular c and b quarks).

Working with limited off-Z-peak statistics (which is *a fortiori* the case with the data obtained from hard initial state radiation $e^+e^- \to f^+f^-\gamma$) we need to provide an efficient way to extract the helicity amplitudes.

In the absence of polarization at LEP, the simplest is to use the angular dependence of the various terms:

$$\begin{aligned} \sigma_{LL}, \sigma_{RR} &\sim (1+\cos\theta)^2 \\ \sigma_{LR}, \sigma_{RL} &\sim (1-\cos\theta)^2 \; . \end{aligned} \tag{2}$$

If no other amplitudes are present (scalar interactions, higher loop corrections) and only these 2 terms contribute, a simple measurement of the integrated forward-backward asymmetry is in principle sufficient. One could also consider projecting on the individual partial waves, (which even neglecting radiative corrections means 3 different polynomials).

Keeping in mind a search for new physics, and the restricted data available, our personal preference consists in <u>projecting out</u> the non-vanishing σ_{ij} by a suitable filter F. The result of this projection is then constituted of contributions from the remaining amplitude at the minimum, radiative corrections (which can be modelled) and any new physics.

For limited sets of data, this procedure seems considerably more stable than a full partial wave analysis. For instance, the $LR+RL$ amplitude (respectively $RR+LL$) are eliminated by the filters

$$\begin{aligned} \tilde{\sigma}_+ = \int_{-\cos\theta max}^{\cos\theta max} \frac{d\sigma}{d\cos\theta} F_+ d\cos\theta &= \sigma_{LL} + \sigma_{RR} \\ &+ \text{rad.corrections} + \text{new physics}, \\ \tilde{\sigma}_- = \int_{-\cos\theta max}^{\cos\theta max} \frac{d\sigma}{d\cos\theta} F_- d\cos\theta &= \sigma_{LR} + \sigma_{RL} \\ &+ \text{rad.corrections} + \text{new physics}. \end{aligned} \tag{3}$$

(For $\cos\theta_{max} = 1$ we have $F_- = 1 - 2\cos\theta$, $F_+ = 1 + 2\cos\theta$).

We will now turn successively to a discussion of the zeroes, the effect of new physics, and finally, comment on the inclusion of radiative corrections, which will be required if off-Z peak data become abundant.

Discussion of the zeroes

We have to consider the amplitudes for the annihilation between an electron of polarization i_1, and an anti-(electron of polarization i_2) into a fermion of pol.j_1 and an anti-(fermion of pol.j_2) (the term of CP conjugate would be more appropriate, since a right-handed positron will be noted $(\bar{e}_L)$).

For instance: $(\bar{e}_L)e_L \to (\bar{\mu}_R)\mu_R$ describes the annihilation of a right-handed positron with a left-handed electron into a right-handed muon and a left-handed antimuon and would be noted: $A_{LL;RR}$.
Gauge vectors respect chirality and, as a result the corresponding A at tree level, will in the above notation always read A_{ij}.

On the contrary, scalar (tensor) interactions have necessarily $i_1 \neq i_2$ and $j_1 \neq j_2$. As a direct consequence they cannot interfere with the above in the massless fermion limit. This has the important consequence that a heavy scalar exchange signal would not benefit from interference with the Z close to the peak; on the contrary, the signal would simply be drowned by the overwhelming Z exchange.

Each of the helicity amplitudes is governed by the $\gamma - Z$ interference, and we have in standard notations, but allowing for an extra gauge boson Z' for later use:

$$\begin{aligned} A^f_{ij}(q^2) &= \frac{-4\pi\bar{\alpha}Q_f}{q^2} + \sqrt{2}G_\mu m_Z^2 \, \frac{g^e_i g^f_j}{q^2 - (m_Z - \frac{i\Gamma_z}{2})^2} + \frac{g'^e_i g'^f_j}{q^2 - (m_{Z'} - \frac{i\Gamma_{Z'}}{2})^2} \\ &= \frac{-4\pi\bar{\alpha}Q_f}{q^2}\left[1 + R^f_{ij}(q^2)\right] \end{aligned} \tag{4}$$

where Q_f if the electric charge of the final state fermion, and g_i are the L and R coupling in the usual conventions. In the standard model:

$$\begin{aligned} g_L &= 2t_{3L} - 2Q\sin^2\theta \ , \\ g_R &= -2Q\sin^2\theta \ , \\ \bar{\alpha} &\equiv \alpha(m_Z^2) \ , \\ \sin^2 2\theta &= \frac{4\pi\bar{\alpha}}{\sqrt{2}G_\mu m_Z^2} \ . \end{aligned} \tag{5}$$

This parametrization uses the best known quantities G_μ, m_Z , $\bar{\alpha}$. Choice of the quantity $\bar{\alpha}$ takes into account the main (electromagnetic) radiative corrections [1].

Destructive interference will occur below the Z peak if

$$(-Q_f)/g_i^l g_j^f > 0 \qquad (6)$$

and above the Z peak otherwise. It turns out in the standard model that for all light fermions $(\mu, \tau, u, d...)$ the destructive interference (leading to near-zeroes) occurs for $\sqrt{s} < m_Z$ in the LL and RR channels (for leptons, this is obvious due to lepton universality), and for $\sqrt{s} > m_Z$ in the $LR + RL$ channels.(see table)

particle type	LL	RR	LR	RL
μ	76.9	80.0	113.1	113.1
u	68.2	80.0	113.1	159.9
d	53.4	80.0	113.1	none

table 1: location of zeroes (in GeV, tree level in weak interactions); in calculations we use $m_Z = 91.188$ GeV, $\sin^2\theta = 0.2311$.

The locations of zeroes are at prominent values of energies: m_W for RR and $\sqrt{2}m_W$ for LR (for any channel), $\sqrt{2}m_W$ for RL for charged leptons.

In practice the projected cross sections $\tilde{\sigma}_+, \tilde{\sigma}_-$ do not reach a zero minimum. This for two reasons:

i) we have (although this is technically beyond the tree level) to take into account the width of the Z, which gives an imaginary part to the sole Z contribution; resulting in a residual cross-section $\sim \Gamma_Z^2$ at the minimum.

ii) selecting definite amplitudes by projection does not allow to distinguish between LL and RR; (eq.2). Since the zeros in $Re(A_{LL})$ and $Re(A_{RR})$ occur at different points, $\tilde{\sigma}_+$ or $\sigma_{LL} + \sigma_{RR}$ only sees the sum of these dips (still a very noticeable effect).

The location of the zero for cross section σ_{ij} is given by the following equation:

$$\frac{m_Z^2}{E_0^2} = 1 - \frac{1}{4s^2c^2}\frac{g_i^e g_j^f}{Q_f} \quad , s^2 \equiv \sin^2\theta \ , \quad c^2 = \cos^2\theta \quad . \tag{7}$$

Their separation would require, in addition to filtering, the identification of one polarization (initial state at SLC or τ polarization at LEP). The same is true for quarks produced in the $LR + RL$ channel above the Z peak.

The only case where we can really approach a unique minimum is for leptons in the $LR + RL$ channel above the Z peak. [2] As we shall see however, the combination of 2 nearby minima still yields a severe suppression of the signal!

Filters

We have advocated above (eq.3) the use of a simple linear filter to project out the unwanted $(LL + RR)$ or $(LR + RL)$ contributions.
As the reader will check easily, this choice of a simple linear filter is insufficient to eliminate for instance a hypothetical scalar component: (A and C_M are defined in eq.11 below)

$$\int_{-\cos\theta_{max}}^{\cos\theta_{max}} S_S \quad F_\pm \quad d\cos\theta = A \cdot C_M \cdot \sigma_S \tag{8}$$

This choice of filtering is however deliberate, for 3 reasons:

- our purpose is to put new physics into evidence, and the way to achieve this is to filter out the unwanted (non-vanishing) component of the known amplitude; namely, below the Z poles we want to filter out the $LR + RL$ part, in order to examine the minimum of the $RR + LL$ part, and put in evidence possible new contributions. (σ_S is just one example, other terms are possible).

[2] If the position of the minimum can be measured with high accuracy this will give additional way to study electroweak radiative corrections.

- radiative corrections can be relatively easily handled in this scheme (see below).

- a more elaborate fit, projecting on all the possible amplitudes (e.g, a Legendre polynomial expansion to order $\cos^2\theta$) proves much less stable then the above simple prescription.

The (differential) tree level cross sections corresponding to eq.4 read:

$$
\begin{aligned}
S_{ij} &= N_C.\frac{\alpha^2}{8s}.2\pi|1+R^f_{ij}|^2.Q_f^2 \\
\frac{d\sigma}{d\cos\theta} &= \frac{(S_{LR}+S_{RL})}{2}(1-\cos\theta)^2+\frac{(S_{RR}+S_{LL})}{2}(1+\cos\theta)^2+S_S \\
\sigma &= \frac{8}{3}\frac{(S_{LR}+S_{RL}+S_{RR}+S_{LL})}{2}+2S_S
\end{aligned}
\tag{9}
$$

where we have added for the sake of further discussion a hypothetical scalar exchange contribution.
The filters introduced in eq.3 are defined so that:

$$
\begin{aligned}
\int_{-\cos\theta_{max}}^{\cos\theta_{max}}(1\pm\cos\theta)^2F_{\pm}d\cos\theta &= \frac{8}{3} \\
\int_{-\cos\theta_{max}}^{\cos\theta_{max}}(1\pm\cos\theta)^2F_{\mp}d\cos\theta &= 0
\end{aligned}
\tag{10}
$$

which yields :

$$
\begin{aligned}
F_{\pm} &= A\,(1\pm B\cos\theta) \\
A &= \frac{2}{C_M(3+C_M^2)} \\
B &= \frac{3+C_M^2}{2C_M^2} \\
C_M &= \cos\theta_{max}
\end{aligned}
\tag{11}
$$

In Figs 1 and 2, we plot the standard model expectation for

$$\frac{\tilde{\sigma}_+^\mu}{\tilde{\sigma}_-^\mu} = \frac{\int \frac{d\sigma}{d\cos\theta}.F_+ d\cos\theta}{\int \frac{d\sigma}{d\cos\theta}.F_- d\cos\theta} \tag{12}$$

for $\sqrt{s} < m_Z$ and $\frac{\tilde{\sigma}_-^\mu}{\tilde{\sigma}_+^\mu}$ for $\sqrt{s} > m_Z$.
While, as previously discussed, the dip in $\sigma_{RR} + \sigma_{LL}$ results from the superposition of 2 nearby minima, σ^μ_{LR+RL} is easier to interpret and is proportional to Γ_Z^2.

We want to stress the relation between filtering and the forward-backward asymmetry A_{FB}. According to the standard definition, we get considering only the contributions in eq. 9 (more complicated terms are expected from radiative corrections)

$$A_{FB} = \frac{3}{4}\,\frac{\sigma_{LL} + \sigma_{RR} - \sigma_{LR} - \sigma_{RL}}{\sigma_{LL} + \sigma_{RR} + \sigma_{LR} + \sigma_{RL} + 2\;\sigma_S} \tag{13}$$

We note first that A_{FB} has only (and by design) little sensitivity to a possible scalar contribution, at the difference of $\tilde{\sigma}_+$ or $\tilde{\sigma}_-$ near to a zero.
Only in the case of pure single vector exchanges (SVE) (like the tree-level standard model) are the two presentations equivalent, with the simple relation:

$$\frac{\tilde{\sigma}_+}{\tilde{\sigma}_-}|_{SVE} = \frac{1 + 4/3\;A_{FB}}{1 - 4/3\;A_{FB}} \tag{14}$$

Windows for new physics.

One of the standard searches [2] is for extra Z' neutral bosons. Such particles would simply add to the existing amplitude, according to eq. 4. In the case of $e^+e^- \to \mu^+\mu^-$ below the Z peak, we want to concentrate on the $LL + RR$ amplitudes. If the Z' couplings respect lepton universality ($g_i'^f = g_i'^e$) it is easy to see that they simply enhance the Z effect, thereby bringing the minima to lower $\sqrt{s}$; the resulting curve dips below the standard one for $\sqrt{s} \ll m_Z$, but raises above it on the rising flank of the Z peak.

The opposite is observed if for instance ($g_i'^f = -g_i'^e$), a situation expected in "horizontal" interactions.

We don't expect spectacular limits to arise for far off-peak data however, since extra gauge boson contributions interfere with the real part of the (γ, Z) amplitudes and are thus widely enforced when this is large: sitting on the minimum of the γ, Z curve minimises this effect, and gain in background reduction is unlikely to compensate for the loss in statistics. In passing however, we note that the analysis of ref. [2] would usefully be extended to non-universal couplings in order to allow both for constructive and destructive interference.

Quite different is the situation for scalar boson exchanges. As stressed before, conservation of chirality prevents their interference wit gauge boson contributions in the massless fermion limit. The best place to look for such a signal is at the minimum (zero) of the gauge contribution. Here, the filter technique has a clear advantage, as the forward-backward asymmetry is rather insensitive to scalars.

In quite a different domain, it is amusing to note that in the $LR + RL$ projection of the $\bar{\mu}\mu$ cross section, the minimum (up to radiative corrections) is proportional to $\Gamma_Z^2(s)$, thus allowing in principle an independent determination of this quantity off the Z peak.

Let us stress as well that this minimum occurs at $E = \sqrt{2} m_W$. So m_W can be measured at LEP 1.5 with the accuracy close to that of m_Z below $2W$ threshold (at least in principle).

The above considerations can obviously be translated (with greater experimental difficulties) to τ leptons (where polarization identification combined with projection would then allow a complete separation of the 4 amplitudes!), and to the quark sector.

We had some hope that the existence of different minima for up and down quarks would allow for an easy separation of these contributions. Unfortunately, as seen from the table, such separation is only quantitative. For the sake of argument, we present in fig (3) the ratio $\tilde{\sigma}_(d)/\tilde{\sigma}_(u)$ which shows the possibility for an enrichment for instance in b or s versus c quarks. Here also, the loss in statistics is such that this method could only be of use as a last ditch effort to improve a sample's purity.

<u>Radiative corrections</u>:

With increasing data, radiative corrections will be needed.
Following standard usage, they are usefully divided in 3 groups:

- electroweak corrections to the propagators and vertices; these are most easily taken into account, and amount essentially to effective (momentum-dependent) coupling constants.

  The largest corrections of this type are of electromagnetic nature and have been taken into account already in eqs. 4,5.

- initial and final state soft radiation; their treatment is usually experiment dependent; we can expect them to factorize so that the ratios $\tilde{\sigma}_+/\tilde{\sigma}_-$ would not be significally affected.

- box diagrams are the most difficult new contributions, since they induce a much more complicated angular ($\cos\theta$) dependence. It seems both difficult and unnecessary to fit higher order polynomials in $\cos\theta$. Instead, we suggest to simply apply the above filters to the full theoretical cross sections using the existing codes [3] and to compare the experimental curves to that result.

Conclusions

We have suggested a presentation of $e^+e^- \to \bar{f}f$ data, in terms of helicity instead of asymmetries, and suggested the use of a specific projection filter to achieve this goal.

This presentation allows for a more transparent interpretation and understanding of the data, in particular where new physics searches is involved.

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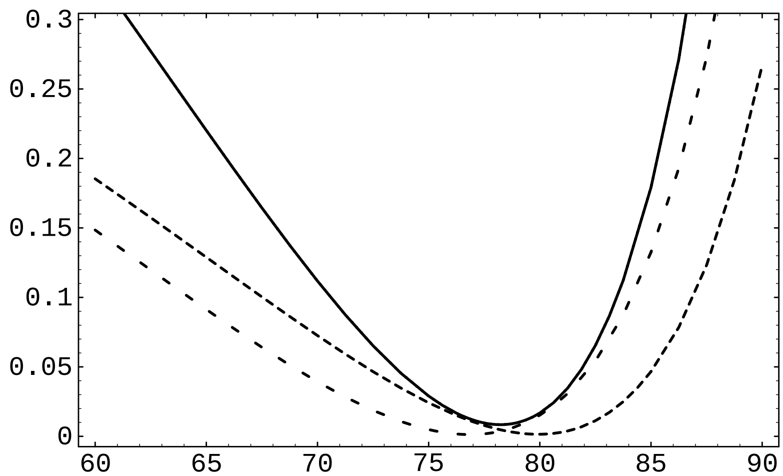

Figure 1: $\frac{\sigma_{LL}}{(\sigma_{LR}+\sigma_{RL})}$ (spaced dashes), $\frac{\sigma_{RR}}{(\sigma_{LR}+\sigma_{RL})}$ (close dashes), $\frac{\sigma_{LL}+\sigma_{RR}}{(\sigma_{LR}+\sigma_{RL})}$ (solid line) as a function of center of mass energy (in GeV) for $e^+e^- \to \mu^+\mu^-$ below the Z peak.

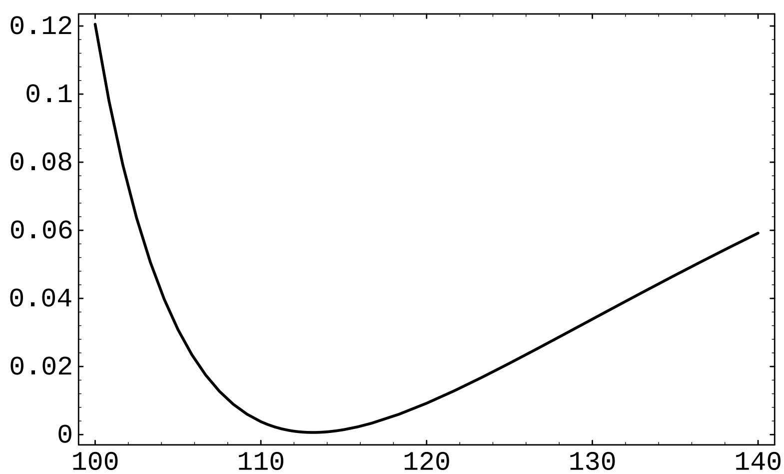

Figure 2: $\frac{(\sigma_{LR}+\sigma_{RL})}{(\sigma_{LL}+\sigma_{RR})}$ as a function of center of mass energy (in GeV) for $e^+e^- \to \mu^+\mu^-$ above the Z peak.

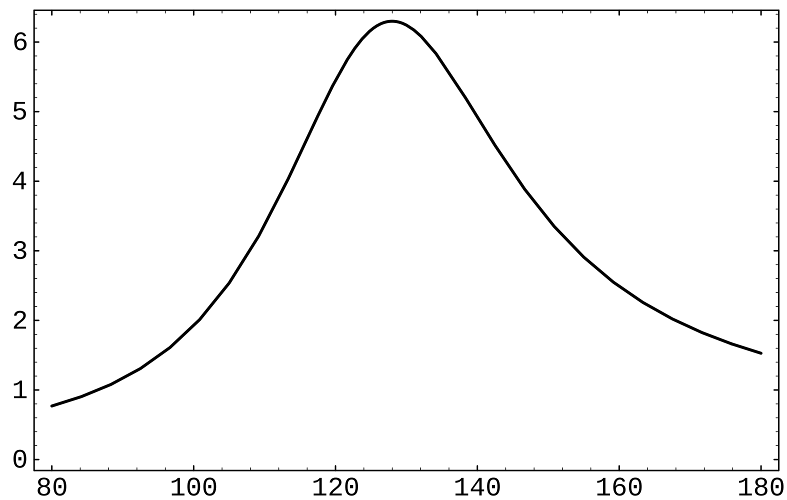

Figure 3: $\frac{\sigma^d_{LR+RL}}{\sigma^u_{LR+RL}}$ as a function of center of mass energy (in GeV); the enhancement of d quarks production with respect to u quarks is clearly seen in this channel.